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ABSTRACT

Recently, significant progress has been made in structural reliability theory and structural optimization theory, respectively. This paper deals with the design problem to find the optimal design when some requirements related to the reliability of the structure are present. Most of the optimization algorithms available require accurate estimates of the gradients. When the reliability constraints are prescribed in terms of the first-order systems reliability index a new efficient method is proposed to evaluate the gradients. A new sequential optimization procedure is proposed which makes use of updated estimates of the Lagrangian multipliers of the reliability-based optimization problems. The procedure is compared with other optimization procedures in an example where a K-joint in a tubular offshore steel jacket structure is considered. The reliability constraints are related to the first-order reliability indices of local stability, punching, yielding and fatigue failure criteria.

RELIABILITY-BASED OPTIMIZATION OF STRUCTURAL ELEMENTS

John Dalsgård Sørensen*

1. INTRODUCTION

During the last 20 years considerable progress has been made in the area of structural reliability theory. Especially the development of the so-called first-order reliability methods (FORM) has been very important [6, 7, 9, 16, 25]. These methods can be used to estimate the reliability of structural elements and very efficient algorithms exist for this purpose. The problem of estimating the reliability of structural systems is much more difficult and only a few practically usable methods exist. Most of these methods assume that the structural system can be modelled by linear elastic perfectly plastic or linear elastic brittle elements [2, 3, 17, 19, 26]. Structures, where fatigue failure is an important failure mode, are examples of structural systems where the element reliability can be estimated by FORM methods but where an estimate of the reliability of the whole structural system requires further research [22, 29].

Also in the field of structural optimization considerable progress has been made in the last few decades [14, 4]. The developments have mainly concentrated on deterministic, classical optimization problems where the constraints signify that the displacements, stresses, etc. must not exceed critical limits. Only a little research has considered the problem of optimum design of structural systems with reliability-based constraints [8, 20, 23]. One of the significant difficulties in solving this type of optimization problems is to estimate the gradients of the reliability constraints, especially when a reliability system is used in the modelling. In this paper an efficient method for determination of the gradients of reliability-based constraints is described. These gradient estimates are used in some optimization procedures, where the mathematical optimization problems are solved using the NLPQL algorithm [21]. NLPQL is a sequential, quadratic optimization algorithm with augmented Lagrangian line search.

In this paper structural elements from an optimization point of view are considered, i.e. only the geometry of a structural element is optimized. Reliability modelling of the structural element is discussed both from an element point of view and from a system point of view.

The optimization prodecures are demonstrated in an example where the geometry of a K-joint in an offshore steel structure is optimized. The constraints require that the reliability against yielding, local buckling, punching and fatigue should be greater than some critical limits.

2. RELIABILITY ANALYSIS OF STRUCTURAL ELEMENTS AND SYSTEMS

The first-order reliability method (FORM) to estimate the structural reliability is briefly summarized. First, a failure element is introduced. A reliability failure element is defined for each possible failure mode, for example yielding of a cross-section or fatigue failure at a critical point in a K-joint in a steel jacket platform. m of the quantities which influence the limit state

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functions for given failure modes are assumed to be modelled as basic stochastic variables X_i, $i = 1, 2, \ldots, m$. These variables can model for example yield strength, wind load, wave load, and geometrical quantities. Each failure element is described by a safety margin

$$M_i' = g_i(X_1, X_2, \ldots, X_m) \tag{1}$$

in terms of the basic variables. $M_i' \leq 0$ corresponds to failure in the i^{th} failure element, and $g_i(\underset{\sim}{x})$ defines the limit state surface for the i^{th} failure element.

The second step in a FORM method is to define a transformation

$$\underset{\sim}{U} = \underset{\sim}{T}(\underset{\sim}{X}) \tag{2}$$

of the vector of basic variables $\underset{\sim}{X}$ into a vector $\underset{\sim}{U}$ of normalized and independent normal variables $\underset{\sim}{U}$. One possible transformation is the Rosenblatt transformation, where $\underset{\sim}{T}$ is defined by, see [16]

$$\begin{aligned} U_1 &= \Phi^{-1}(F_1(X_1)) \\ U_2 &= \Phi^{-1}(F_2(X_2|X_1)) \\ &\vdots \\ U_m &= \Phi^{-1}(F_m(X_m|X_1, X_2, \ldots, X_{m-1})) \end{aligned} \tag{3}$$

where Φ denotes the standardized normal distribution function and $F_i(x_i|x_1, x_2, \ldots, x_{i-1})$ is the conditional distribution function for X_i given $X_1 = x_1, X_2 = x_2, \ldots, X_{i-1} = x_{i-1}$.

In the third step the reliability index β_i for failure element i is determined as the shortest distance from the origin to the failure surface $g_i(\underset{\sim}{x}) = g_i(\underset{\sim}{T}^{-1}(\underset{\sim}{u})) = 0$ in the $\underset{\sim}{u}$-space:

$$\beta_i = \min_{g_i(\underset{\sim}{T}^{-1}(\underset{\sim}{u})) = 0} |\underset{\sim}{u}| \tag{4}$$

Generally, β_i has to be determined iteratively, e.g. by using an optimization algorithm. The point $\underset{\sim}{u}^*$ on the failure surface closest to the origin is the design point. The corresponding point in the $\underset{\sim}{x}$-space is $\underset{\sim}{x}^* = \underset{\sim}{T}^{-1}(\underset{\sim}{u}^*)$. The safety margin M_i' is now linearized in the design point:

$$M_i' \cong M_i = \beta_i + \underset{\sim}{\alpha}_i^T \underset{\sim}{U} \tag{5}$$

where

$$\underset{\sim}{\alpha}_i = \frac{-\nabla g_i(T^{-1}(\underset{\sim}{u}^*))}{|\nabla g_i(T^{-1}(\underset{\sim}{u}^*))|} \tag{6}$$

The $\underset{\sim}{\alpha}_i$ coefficients are measures of the sensitivity of β_i with respect to changes in $\underset{\sim}{u}$.

Next, structural systems are considered. If there is h possible failure elements and the structure is defined to fail if one of these elements fails, then the probability of failure P_f^S of the structure is determined as the probability of failure of a series system:

$$P_f^S = P(\{M_1' \leq 0\} \cup \{M_2' \leq 0\} \cup \ldots \cup \{M_h' \leq 0\})$$

$$\cong P(\{M_1 \leq 0\} \cup \{M_2 \leq 0\} \cup \ldots \cup \{M_h \leq 0\})$$

$$= 1 - \Phi_h(\underset{\sim}{\beta} ; \underset{\sim}{\rho}) \tag{7}$$

where Φ_h is the h-dimensional normalized normal distribution function. $\underset{\sim}{\rho}$ is the correlation coefficient matrix where

$$\rho_{ij} = \underset{\sim}{\alpha}_i^T \underset{\sim}{\alpha}_j \tag{8}$$

The reliability index β_S for the series system is defined as

$$\beta_S = -\Phi^{-1}(P_f^S) \tag{9}$$

If failure of the structural system is defined as the event that all h failure elements have to fail, then the probability of failure P_f^P is determined as the probability of failure of a parallel system:

$$P_f^P = P(\{M_1' \leq 0\} \cap \{M_2' \leq 0\} \cap \ldots \cap \{M_h' \leq 0\})$$

$$\cong P(\{M_1 \leq 0\} \cap \{M_2 \leq 0\} \cap \ldots \cap \{M_h \leq 0\})$$

$$= \Phi_h(-\underset{\sim}{\beta} ; \underset{\sim}{\rho}) \tag{10}$$

The reliability index β_P for a parallel system is defined as

$$\beta_P = -\Phi^{-1}(P_f^P) \tag{11}$$

Finally, the failure event that k out of h failure elements fail is considered. It is assumed that H significant failure modes of k failure elements are identified, see [2, 3, 17, 19, 26]. A generalization of the first-order mothods for series and parallel systems gives the following estimate of the probability of failure P_f^G:

$$P_f^G \cong 1 - \Phi_H(\underset{\sim}{\beta}^* ; \underset{\sim}{\rho}^*) \tag{12}$$

where

$$\beta_i^* = \beta_i^*(\underset{\sim}{\epsilon})\Big|_{\underset{\sim}{\epsilon} = \underset{\sim}{0}} = -\Phi^{-1}(\Phi_k(-\underset{\sim}{\bar{\beta}}_i + \underset{\sim}{\bar{\alpha}}_i \underset{\sim}{\epsilon}\, ; \underset{\sim}{\bar{\rho}}_i))\Big|_{\underset{\sim}{\epsilon} = \underset{\sim}{0}} = -\Phi^{-1}(\Phi_k(-\underset{\sim}{\bar{\beta}}_i\, ; \underset{\sim}{\bar{\rho}}_i)) \tag{13}$$

$$\rho_{ij}^* = \underset{\sim}{\alpha}_i^{*T} \underset{\sim}{\alpha}_j^* \tag{14}$$

$$\underset{\sim}{\alpha}_i^* = -\frac{\nabla \beta_i^*(\underset{\sim}{0})}{|\nabla \beta_i^*(\underset{\sim}{0})|} \tag{15}$$

$\underset{\sim}{\bar{\beta}}_i = (\bar{\beta}_{i(1)}, \ldots, \bar{\beta}_{i(k)})$ are the reliability indices of the k failure elements in the i^{th} failure mode, $\underset{\sim}{\bar{\rho}}_i$ is the corresponding correlation coefficient matrix and $\underset{\sim}{\bar{\alpha}}_i$ a matrix with the corresponding $\underset{\sim}{\alpha}_i$ vectors.

The reliability index β_S is defined as

$$\beta_G = -\Phi^{-1}(P_f^G) \tag{16}$$

For a failure element the asymptotic result for $\beta_i^e = -\Phi^{-1}(P(M_i' \leq 0))$ is derived, [12]

$$\beta_i^e \sim \beta_i \quad , \quad \beta_i = |\underset{\sim}{u}_i^*| \rightarrow \infty \tag{17}$$

3. FORMULATION OF RELIABILITY-BASED OPTIMIZATION PROBLEMS

In classical deterministic structural optimization the optimization problem is usually formulated with the structural weight as objective function and constraints ensuring that the stresses, displacements, etc. do not exceed code established critical values, [28].

The optimization variables are denoted $\underset{\sim}{t} = (t_1, t_2, \ldots, t_n)$ and are usually geometrical quantities. The variables are often divided into two groups, namely coordinate variables used in shape-optimization and cross-sectional variables. In this paper only the latter group is considered.

In structural optimization many different objective functions have been proposed, e.g. the cost (total and/or initial) of the structure and utility functions. However, difficulties in assessing monetary values have caused that usually the structural weight is used as objective function.

In reliability-based structural optimization a number of formulations can be used. First, consider the case where the reliability of the structure is measured by a systems reliability index, e.g. β_S. A reliability-based structural optimization problem can then be formulated as

F1 minimize $F(\underset{\sim}{t})$

$$\text{s.t.} \quad \beta_S(\underset{\sim}{t}) \geq \beta_S^0 \qquad \underset{\sim}{t}^\ell \leq \underset{\sim}{t} \leq \underset{\sim}{t}^u \tag{18}$$

where $F(\underset{\sim}{t})$ is the objective function and β_S^0 some target reliability index. $\underset{\sim}{t}^\ell$ and $\underset{\sim}{t}^u$ are lower and upper bounds for $\underset{\sim}{t}$.

This optimization problem is in general a non-linear non-convex constrained optimization problem. In [24] it is shown that for structures with relatively high reliability indices the problem tends to be convex. Numerical procedures to solve the mathematical optimization problem (18) are discussed later.

The optimization problem can also be formulated as

$$\begin{aligned} \text{F2} \quad & \text{minimize} \quad -\beta_S(\underset{\sim}{t}) \\ & \text{s.t.} \qquad F(\underset{\sim}{t}) \geqslant F^0 \\ & \qquad\qquad \underset{\sim}{t}^{\ell} \leqslant \underset{\sim}{t} \leqslant \underset{\sim}{t}^{u} \end{aligned} \tag{19}$$

or as a multicriteria optimization problem, [5, 15], where both of the conflicting objective functions in (18) and (19) are to be minimized simultaneously

$$\begin{aligned} \text{F3} \quad & \text{minimize} \quad F(\underset{\sim}{t}) \\ & \qquad\qquad\quad -\beta_S(\underset{\sim}{t}) \\ & \text{s.t.} \qquad \underset{\sim}{t}^{\ell} \leqslant \underset{\sim}{t} \leqslant \underset{\sim}{t}^{u} \end{aligned} \tag{20}$$

In general there is no unique solution to (20), but in the space of the functions $-\beta_S$ and F a set of possible solutions is defined. The advantage of using (20) is that the designer can introduce social and ethical elements in the design process and that the different and conflicting requirements can be dealt with more effectively.

The solution to (20) which maximizes the designer's utility is often chosen from the Pareto optimum set, [5, 15]. Generally, a point $\underset{\sim}{t}^*$ in the feasible region is Pareto optimal if no decrease can be obtained in any of the objective functions without causing a simultaneous increase in (at least one of) the other objective function(s). However, in order to determine the Pareto optimum set a number of optimization problems of the form (18) or (19) generally has to be solved. Therefore, it can be very computer-time consuming to determine the Pareto points.

Alternatively, to determine Pareto optimal points a sensitivity study can be performed. The sensitivity of the structural weight F from changes in the target reliability index β_S^0 can be found by considering the Lagrangian function L of (18):

$$L(\underset{\sim}{t}, \underset{\sim}{\lambda}) = F(\underset{\sim}{t}) - \lambda_0(\beta_S(\underset{\sim}{t}) - \beta_S^0) - \sum_{i=1}^{n} \lambda_i(t_i - t_i^{\ell}) - \sum_{i=1}^{n} \lambda_{n+1}(t_i^u - t_i) \tag{21}$$

where $\underset{\sim}{\lambda}$ is the vector of Lagrangian multipliers. The necessary conditions for $\underset{\sim}{t}^*$ to be an optimal point and $\underset{\sim}{\lambda}^*$ to be the corresponding multipliers require that (Kuhn-Tucker conditions)

$$\begin{cases} \beta_S(\underset{\sim}{t}^*) \geq \beta_S^0 & (22) \\ t_i^\ell \leq t_i^* \leq t_i^u \quad , \quad i = 1, 2, \ldots, n & (23) \end{cases}$$

$$\begin{cases} \lambda_0^*(\beta_S(t^*) - \beta_S^0) = 0 \quad , \quad \lambda_0^* \geq 0 & (24) \\ \lambda_i^*(t_i^* - t_i^\ell) = 0 \quad , \quad \lambda_i^* \geq 0 \quad , \quad i = 1, 2, \ldots, n & (25) \\ \lambda_{n+i}^*(t_i^u - t_i^*) = 0 \quad , \quad \lambda_{n+i}^* \geq 0 \quad , \quad i = 1, 2, \ldots, n & (26) \end{cases}$$

$$\frac{\partial F(t^*)}{\partial t_i} - \lambda_0^* \frac{\partial \beta_S(t^*)}{\partial t_i} - \lambda_i^* + \lambda_{n+i}^* = 0 \quad , \quad i = 1, 2, \ldots, n \tag{27}$$

From (24) it is seen that if (22) is active and at least one of the constraints in (23) is not active then

$$\frac{\partial F(t^*)}{\partial \beta_S(t^*)} = \lambda_0^* \tag{28}$$

i.e. λ_0^* is a measure of the sensitivity of F with respect to β_S^0.

Secondly, we consider the case where the reliability of the structure is measured by the element reliability indices β_i, i = 1, 2, . . . , h. A reliability-based structural optimization problem can then be formulated as

F4 minimize $F(\underset{\sim}{t})$

$$\begin{aligned} \text{s.t.} \quad & \beta_i(t) \geq \beta_i^0 \quad , \quad i \quad , \quad i = 1, 2, \ldots, h & (29) \\ & t_i^\ell \leq t_i \leq t_i^u \quad , \quad i = 1, 2, \ldots, n & \end{aligned}$$

where $\underset{\sim}{\beta}^0$ is the minimum permissible reliability indices.

As the above formulations (29) is a non-linear and non-convex optimization problem. Compared to (18) the number of constraints is increased by h − 1 but the element reliability indices are generally much easier to evaluate than the systems reliability index. In some cases a mixed formulation of (18) and (29) is most natural.

In many cases it can be necessary to augment the reliability-based structural optimization problems (18) or (29) with deterministic constraints relating the optimization variables. These constraints can be handled as usual constraints in structural optimization.

4. ESTIMATION OF GRADIENTS OF RELIABILITY INDICES

Accurate estimates of the gradients of the reliability indices $\beta_i(\underset{\sim}{t})$ are generally required in order to achieve convergence of mathematical optimization algorithms when applied to reliability-based optimization problems, [24]. One possibility is to use numerical differentiation. But if n is large numerical differentiation can be very time-consuming, especially for systems reliability indices. However, in [12] an efficient estimate of the gradient of the element reliability index β_i has been proposed. The limit state function for failure element i is written

$$g_i(\underset{\sim}{u}, \underset{\sim}{t}) = 0 \tag{30}$$

where $\underset{\sim}{u}$ is a point in the normal variable space R^m and $\underset{\sim}{t}$ is the vector of optimization variables, here geometrical quantities. If $\underset{\sim}{u}^*$ is the design point then the reliability index is $\beta_i = |\underset{\sim}{u}^*|$. Then it follows that

$$\frac{\partial \beta_i}{\partial t_j} = \frac{1}{|\nabla_{\underset{\sim}{u}}\, g_i(\underset{\sim}{u}^*, \underset{\sim}{t})|}\;\frac{\partial g_i(\underset{\sim}{u}^*, \underset{\sim}{t})}{\partial t_j} \quad , \quad j = 1, 2, \ldots, n \tag{31}$$

where the gradient $\nabla_{\underset{\sim}{u}}\, g_i(\underset{\sim}{u}^*, \underset{\sim}{t})$ is the vector of derivatives of g_i with respect to $\underset{\sim}{u}$ evaluated at $(\underset{\sim}{u}^*, \underset{\sim}{t})$. This gradient is determined when calculating β_i and $\partial g_i/\partial t_i$ is easily calculated. In [12] it is shown that asymptotically for $\beta_i \to \infty$ (31) gives the gradient of $\beta_i = -\Phi^{-1}(P(g_i(\underset{\sim}{U}, \underset{\sim}{t}) \leq 0))$. Now consider the series systems reliability index β_S, (9)

$$\beta_S(\underset{\sim}{t}) = -\Phi^{-1}(1 - \Phi_h(\underset{\sim}{\beta}(\underset{\sim}{t})\, ; \underset{\sim}{\rho}(\underset{\sim}{t}))) \tag{32}$$

The derivative of $\beta_S(\underset{\sim}{t})$ is

$$\frac{\partial \beta_S(\underset{\sim}{t})}{\partial t_i} = \frac{1}{\varphi(\beta_S)}\left[\sum_{j=1}^{h} \frac{\partial \Phi_h}{\partial \beta_j}\frac{\partial \beta_j}{\partial t_i} + 2\sum_{j=1}^{h}\sum_{k=1}^{j-1} \frac{\partial \Phi_h}{\partial \rho_{jk}}\frac{\partial \rho_{jk}}{\partial t_i}\right] \tag{33}$$

where $\varphi(\cdot)$ is the normal density function.

$\partial \beta_j/\partial t_i$ can be determined by (31). $\partial \rho_{jk}/\partial t_i$ is determined approximately by

$$\frac{\partial \rho_{jk}}{\partial t_i} = -\frac{1}{|\nabla g_k|\,\beta_j}\sum_{\ell=1}^{m}\frac{\partial u_{j\ell}}{\partial t_i}\frac{\partial g_k}{\partial u_{k\ell}} - \frac{1}{\beta_j}\frac{\partial \beta_j}{\partial t_i}\rho_{jk}$$

$$-\frac{1}{|\nabla g_j|\,\beta_k}\sum_{\ell=1}^{m}\frac{\partial u_{k\ell}}{\partial t_i}\frac{\partial g_j}{\partial u_{j\ell}} - \frac{1}{\beta_k}\frac{\partial \beta_k}{\partial t_i}\rho_{jk}$$

$$\cong \left(\frac{\rho_{jk}}{|\rho_{jk}|} - \rho_{jk}\right)\left(\frac{1}{\beta_j}\frac{\partial \beta_j}{\partial t_i} + \frac{1}{\beta_k}\frac{\partial \beta_k}{\partial t_i}\right) \tag{34}$$

where $u_{j\ell}$ is the ℓth coordinate of the design point of the jth safety margin. The approximation in (34) becomes negligible as $|\rho_{jk}| \to 1$. When ρ_{jk} is small the error introduced can be significant, but in this case the influence of ρ_{jk} on Φ_h is negligible, see later, so (34) is considered to be a reasonable approximation.

The derivatives of Φ_h with respect to β_j and ρ_{jk} can be written as

$$\frac{\partial \Phi_h}{\partial \beta_j} = \Phi_{h-1}(\underset{\sim}{\beta}'; \underset{\sim}{\rho}')\varphi(\beta_j) \tag{35}$$

$$\frac{\partial \Phi_h}{\partial \rho_{jk}} = \Phi_{h-2}(\underset{\sim}{\beta}''; \underset{\sim}{\rho}'')\varphi_2(\beta_j, \beta_k; \rho_{jk}) \tag{36}$$

where $\varphi_2(\cdot)$ is the two-dimensional normal density function. Expressions for $\underset{\sim}{\beta}'$, $\underset{\sim}{\rho}'$, $\underset{\sim}{\beta}''$ and $\underset{\sim}{\rho}''$ are given in the appendix (equations (A9), (A10), (A17), and (A18)).

If h is relatively large (> 5) the computational work with estimation of (35) and (36) is of the same magnitude as when simple numerical differentiation of Φ_h is used. However, based on (35) simple approximations to $\partial\beta_S/\partial\beta_j$ can be determined. From (35) and upper bound is

$$\frac{\partial \beta_S}{\partial \beta_j} = \frac{1}{\varphi(\beta_S)} \frac{\partial \Phi_h}{\partial \beta_j} \leqslant \frac{\varphi(\beta_j)}{\varphi(\beta_S)} \tag{37}$$

This estimate of $\partial\beta_S/\partial\beta_j$ can be expected to be satisfactory when all $\beta_j \gtrsim 3$ and all $\rho_{ij} \lesssim 0.4$. A more accurate estimate of $\partial\beta_S/\partial\beta_j$ can be obtained by including an approximation to $\Phi_{h-1}(\underset{\sim}{\beta}'; \underset{\sim}{\rho}')$. If h_S is the number of significant reliability indices in $\underset{\sim}{\beta}'$ (for example all reliability indices in the interval $[-\infty; 3.]$) and these are collected in $\underset{\sim}{\beta}'_S$, then $\partial\beta_S/\partial\beta_j$ can be approximated by

$$\frac{\partial \beta_S}{\partial \beta_j} \sim \Phi_{h_S}(\underset{\sim}{\beta}'_S; \underset{\sim}{\bar{\rho}}) \frac{\varphi(\beta_j)}{\varphi(\beta_S)} \tag{38}$$

where

$$\bar{\rho}_{k\ell} = \bar{\rho} = \frac{1}{h_S(h_S-1)} \sum_{i=1}^{h_S} \sum_{\substack{\ell=1 \\ \ell \neq i}}^{h_S} \rho'_{S_{i\ell}} \quad , \quad k \neq \ell \tag{39}$$

is the average correlation coefficient of the correlation coefficients in the correlation coefficient matrix $\underset{\sim}{\rho}'_S$ corresponding to $\underset{\sim}{\beta}'_S$. $\Phi_{h_S}(\underset{\sim}{\beta}'_S; \underset{\sim}{\bar{\rho}})$ is calculated by, [13]

$$\Phi_{h_S}(\underset{\sim}{\beta}'_S; \underset{\sim}{\bar{\rho}}) = \int_{-\infty}^{\infty} \varphi(t) \prod_{i=1}^{h_S} \Phi\left(\frac{\beta'_{S_i} - \sqrt{\bar{\rho}}\, t}{\sqrt{1-\bar{\rho}}}\right) dt \tag{40}$$

In order to evaluate the upper bound (37) when applied to gradient determination and to compare the magnitudes of $\partial\beta_S/\partial\beta_i$ and $\partial\beta_S/\partial\rho_{ij}$ we consider the case where all reliability indices are equal

$$\beta_1 = \beta_2 = \ldots = \beta_h = \beta$$

and where all correlation coefficients are equal

$$\rho_{ij} = \rho \quad , \quad i,j = 1, 2, \ldots, h \quad , \quad i \neq j$$

In this case the series systems reliability index β_S can be calculated using (40):

$$\beta_S = -\Phi^{-1}\left(\int_{-\infty}^{\infty} \left(1-\Phi\left(\frac{\beta-\sqrt{\rho}\,x}{\sqrt{1-\rho}}\right)\right)^h \varphi(x)dx\right) \tag{41}$$

The derivatives of β_S are determined using (35) and (36). $\Phi_{h-1}(\underset{\sim}{\beta}'; \underset{\sim}{\rho}')$ and $\Phi_{h-2}(\underset{\sim}{\beta}''; \underset{\sim}{\rho}'')$ are easily calculated using (40) because all elements in $\underset{\sim}{\beta}'$ are equal, all elements in $\underset{\sim}{\beta}''$ are equal, all correlation coefficients in $\underset{\sim}{\rho}'$ are equal and all correlation coefficients in $\underset{\sim}{\rho}''$ are equal. Therefore, (38) is exact in this case.

In figures 1 - 3 $\partial\beta_S/\partial\beta_i$ and $\partial\beta_S/\partial\rho_{ij}$ are shown as functions of h, β, and ρ.

From figure 1 it is seen that $\partial\beta_S/\partial\rho_{ij} \ll \partial\beta_S/\partial\beta_i$ for $h > 10$ and that the upper bound (37) overestimates $\partial\beta_S/\partial\beta_i$ by a factor of about two for $h > 10$. For small values of h the upper bound (37) is rather good. From figure 2 it is seen that $\partial\beta_S/\partial\rho_{ij} \ll \partial\beta_S/\partial\beta_i$ for $\beta \gtrsim 4$. Further it appears that the overestimation of $\partial\beta_S/\partial\beta_i$ by (37) decreases when β increases. The influence of the correlation coefficient is shown in figure 3. The relative importance of $\partial\beta_S/\partial\rho_{ij}$ compared with $\partial\beta_S/\partial\beta_i$ is seen to increase with ρ. When ρ approaches 1 the upper bound (37) is seen to be unsatisfactory, in fact when $\rho = 1$

$$\frac{\partial\rho_S}{\partial\beta_i} = \frac{\varphi(\beta)}{\varphi(\beta_S)}\,\Phi_{h-1}(\underset{\sim}{0}\,;\underset{\sim}{R}) \tag{42}$$

where all correlation coefficients in $\underset{\sim}{R}$ are 0.5.

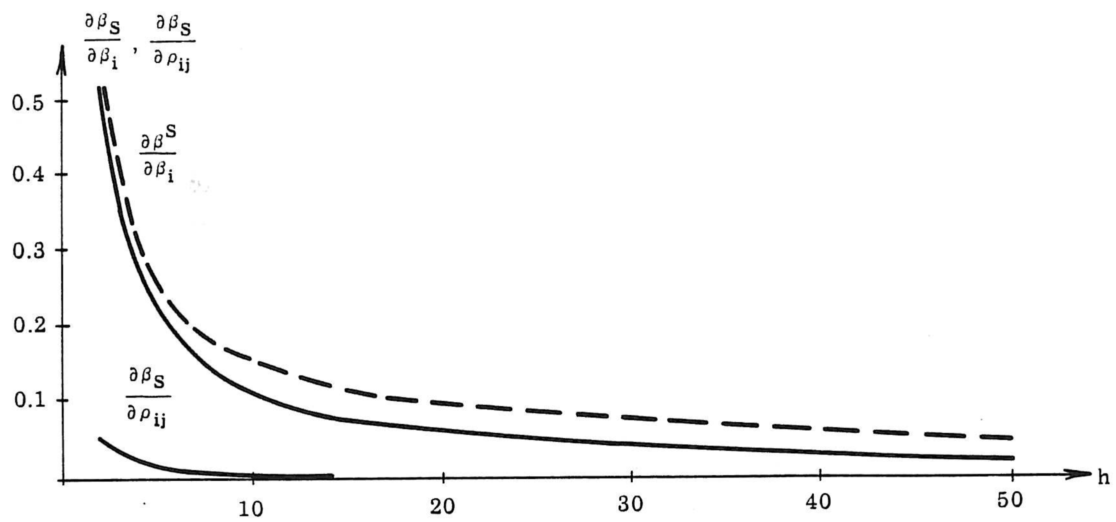


Figure 1. Derivatives of β_S as functions of h for $\beta = 3$ and $\rho = 0.5$. The unbroken lines show the exact derivatives evaluated using (35) and (36) and the broken line shows the upper bound (37).

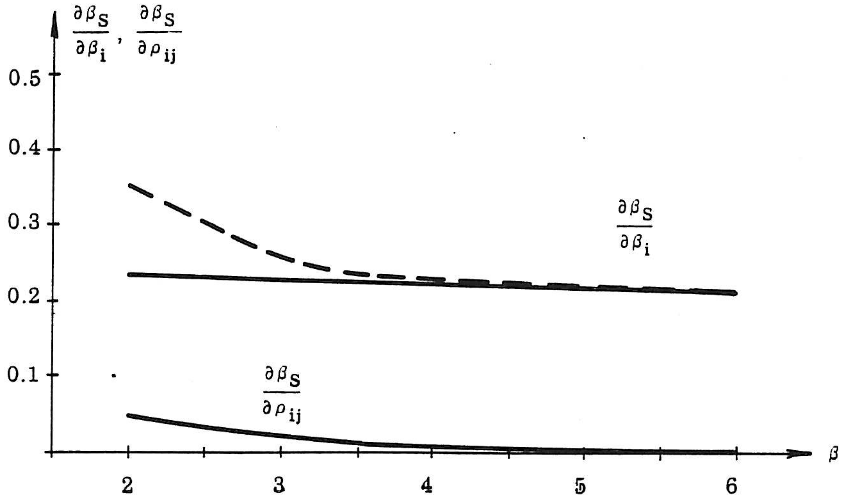


Figure 2. Derivatives of β_S as functions of β for h = 5 and ρ = 0.5. The unbroken lines show the exact derivatives evaluated using (35) and (36) and the broken line shows the upper bound (37).

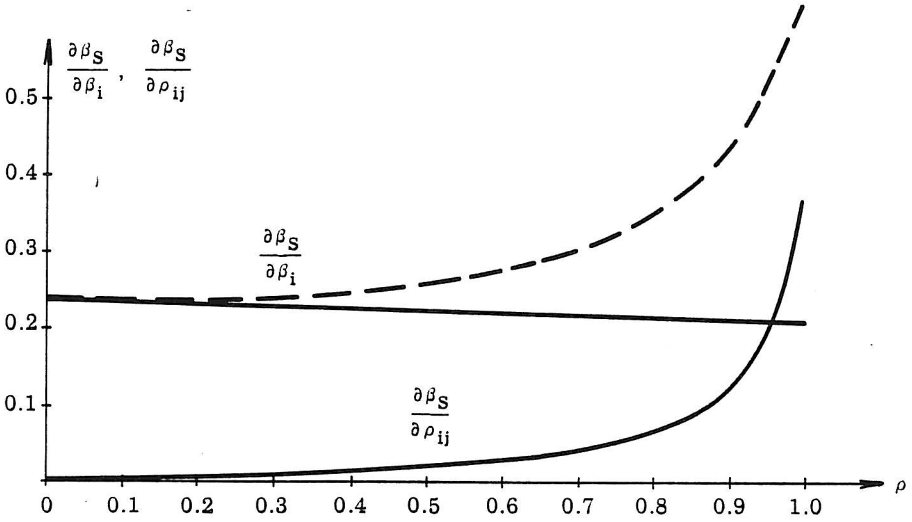


Figure 3. Derivatives of β_S as functions of ρ for h = 5 and β = 3. The unbroken lines show the exact derivatives evaluated using (35) and (36) and the broken line shows the upper bound (37).

For common structural systems the level of the reliability indices β_i is usually greater than 3-4, the correlation moderate ($\rho_{ij} < 0.5$) and the number of significant failure modes small ($h < 10$). From(34) it appears that $o(\partial \rho_{jk}/\partial t_i) = (1/\beta_i)\, o(\min\{\partial \beta_j/\partial t_i\,,\ \partial \beta_k/\partial t_i\})$. From this and from the above figures it seems in most cases to be reasonable to evaluate $\partial \beta_S/\partial t_i$ by one of the following two approximations corresponding to (37) and (38)

$$\frac{\partial \beta_S}{\partial t_i} \sim \frac{1}{\varphi(\beta_S)} \sum_{j=1}^{h} \varphi(\beta_j) \frac{\partial \beta_j}{\partial t_i} \tag{43}$$

$$\frac{\partial \beta_S}{\partial t_i} \sim \frac{1}{\varphi(\beta_S)} \sum_{j=1}^{h} \Phi_{h_S}(\underset{\sim}{\beta}'_S\,; \underset{\sim}{\bar{\rho}})\varphi(\beta_j) \frac{\partial \beta_j}{\partial t_i} \tag{44}$$

With the same assumptions corresponding approximations can be used when the reliability model is a parallel system or a series system of parallel systems. In the latter case the reliability index of the structure is estimated by (12) - (16). The approximation corresponding to (43) becomes

$$\frac{\partial \beta_G}{\partial t_i} \sim \frac{1}{\varphi(\beta_G)} \sum_{j=1}^{H} \sum_{\ell=1}^{k} \varphi(\bar{\beta}_{j\ell}) \prod_{\substack{s=1 \\ s \neq \ell}}^{k} \Phi(-\bar{\beta}_{js}) \frac{\partial \bar{\beta}_{j\ell}}{\partial t_i} \tag{45}$$

In (45) the approximation $\Phi_n(-\beta ; \rho) \sim \prod_{i=1}^{n} \Phi(-\beta_i)$ is used.

Above the derivatives of the reliability index β with respect to the geometrical quantities are estimated. Analogously, derivatives with respect to parameters p in the distribution functions of the stochastic variables X can be found, as shown in [12]. The transformation (2) is written

$$U = T(X , p) \tag{46}$$

The derivatives of β_i and β_S then become, see (31), (43), and (44)

$$\frac{\partial \beta_i}{\partial p_j} = \frac{1}{\beta_i} \sum_{\ell=1} u_\ell^* \frac{\partial T_\ell(x^*, p)}{\partial p_j} \tag{47}$$

$$\frac{\partial \beta_S}{\partial p_j} \sim \frac{1}{\varphi(\beta_S)} \sum_{i=1}^{h} \varphi(\beta_i) \frac{\partial \beta_i}{\partial p_j} \tag{48}$$

$$\frac{\partial \beta_S}{\partial p_j} \sim \frac{1}{\varphi(\beta_S)} \sum_{i=1}^{h} \Phi_{h_S}(\beta_S' ; \bar{\rho}) \varphi(\beta_i) \frac{\partial \beta_i}{\partial p_j} \tag{49}$$

where $x^* = T^{-1}(u^*, p)$ is the design point in the basic variable coordinate system.

5. OPTIMIZATION PROCEDURES

In principle the reliability-based structural optimization problems (18) and (29) can be solved using a standard mathematical optimization algorithm. But such a procedure can be expected to be rather computer time consuming, because it will require many expensive reliability analyses. Therefore, alternative procedures are described in this section.

Some authors (e.g. [20]) have suggested that optimization problems of the type (18) and (29) are solved using a method based on a sequence of linearized problems. The linearized problems are easily solved using standard linear programming algorithms, but convergence to a feasible point cannot be quaranteed neither for problems with only one non-linear constraint (18), nor for problems with multiple constraints (29).

In the optimization procedures proposed in the following the general non-linear programming algorithm NLPQL [21], is used. This algorithm is a very effective method where each iteration consists of two steps. The first step is determination of a search direction by solving a quadratic optimization problem formed by a quadratic approximation of the Lagrangian function of the non-linear problem and a linearization of the constraints at the current design point. The second step is a line search with an augmented Lagrangian merit function. Instead of NLPQL any general non-linear programming algorithm can be used in principle.

First consider problem (29) where the element reliability indices form the constraints.

Procedure PE1

(29) is solved directly by using NLPQL. Gradients of the objective function are calculated by finite differences and gradients of constraints are evaluated using (31).

Next the systems reliability index based problem (18) is considered. Three procedures are proposed:

Procedure PS1

(18) is solved directly by using NLPQL. Gradients are calculated by finite differences.

Procedure PS2

(18) is solved directly by using NLPQL. Gradient of objective function is calculated by finite differences and the gradient of the systems reliability index constraint is evaluated using (31) and the approximation (43).

Procedure PS3

(18) is solved by using a procedure like PS2, but using the approximation (44) instead of (43).

Procedure PS4

In order to reduce the number of systems reliability index evaluations a sequential procedure is proposed where each step consists of solving an optimization problem with element reliability indices as constraints.

The procedure is shown in figure 4, where the steps are:

1)

Input of data and initialising of β_i^j, i = 1, . . . , h.

2)

Solution of a problem equivalent to (29) using NLPQL and gradients calculated by (31).

3)

Evaluation of the systems reliability index $\beta_S(\underset{\sim}{t}^j)$ using for example Hohenbichler's approximation [11].

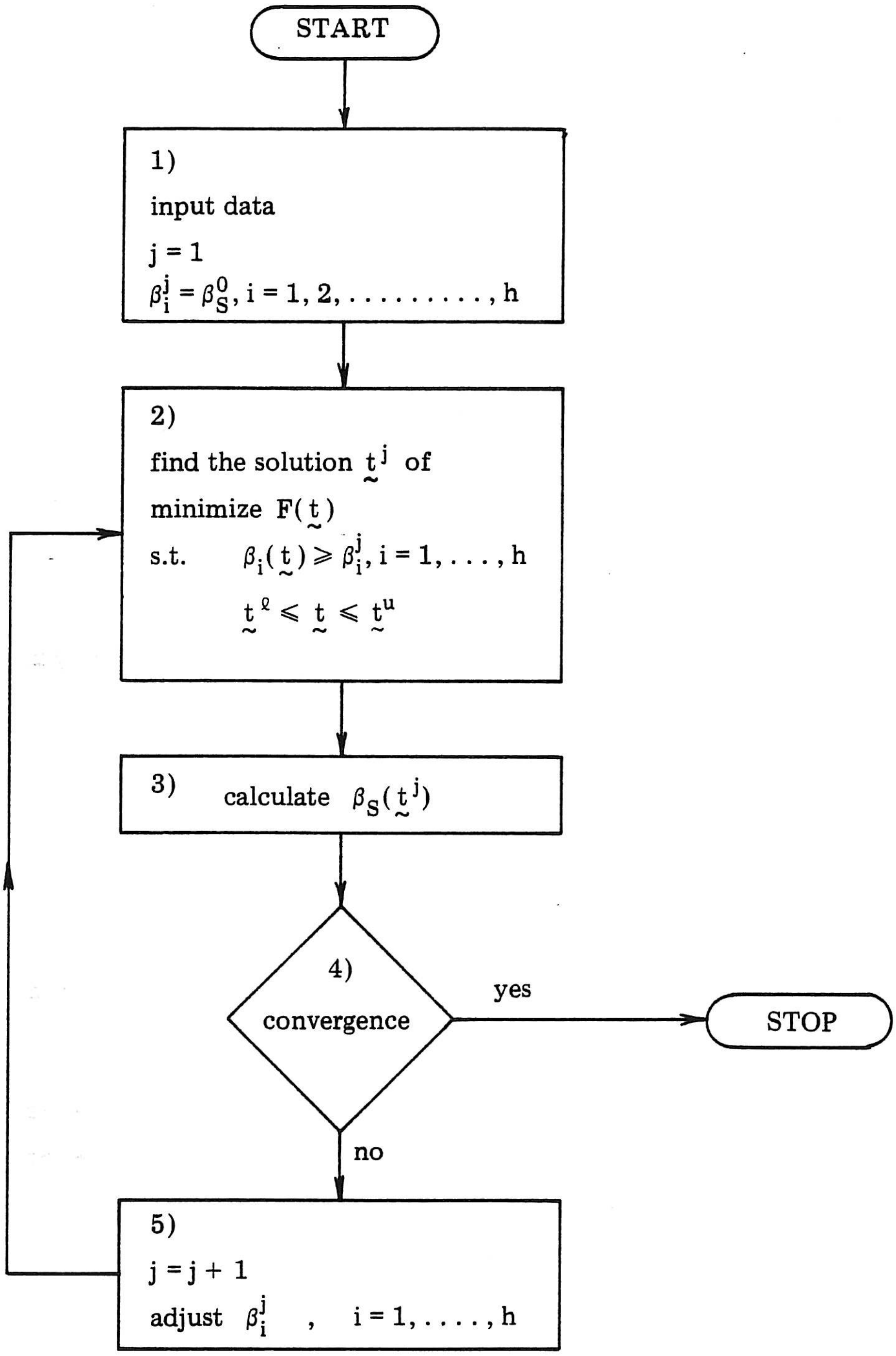


Figure 4. Flow chart for optimization procedure PS4.

4)

The iteration stops if the following two criteria are fulfilled:

a) $|\beta_S^0 - \beta_S(\underset{\sim}{t}^j)| \leqslant \epsilon$

b) $\left| \dfrac{F(\underset{\sim}{t}^j) - F(\underset{\sim}{t}^{j-1})}{F(\underset{\sim}{t}^j)} \right| \leqslant \epsilon$

where ϵ is a constant, e.g. 0.001.

5)

The critical reliability indices β_i^j, $i = 1, \ldots, h$ are adjusted using the Lagrange multipliers from the solution of step 2. Using (43) we can write

$$\Delta\beta_S \cong \frac{1}{\varphi(\beta_S)} \sum_I \varphi(\beta_i)\, \Delta\beta_i \qquad (50)$$

where I is the set of active constraints of element reliability indices. The Lagrange function of the optimization problem in step 2 is equivalent to (21):

$$L(\underset{\sim}{t}, \underset{\sim}{\lambda}) = F(\underset{\sim}{t}) - \sum_{i=1}^{h} \lambda_i(\beta_i(\underset{\sim}{t}) - \beta_i^j) - \sum_{i=1}^{n} \lambda_{h+i}(t_i - t_i^\ell) - \sum_{i=1}^{n} \lambda_{h+n+i}(t_i^u - t_i) \qquad (51)$$

The Kuhn-Tucker conditions, see (22) - (27) then suggest the approximation

$$\Delta\beta_i = \frac{\lambda_\ell}{\lambda_i} \Delta\beta_\ell \quad , \quad \ell, i \in I \qquad (52)$$

(50) then gives the adjusting rule

$$\beta_i^j = \begin{cases} \beta_i^{j-1} + \dfrac{1}{\lambda_i} \dfrac{\varphi(\beta_S^{j-1})\Delta\beta_S^{j-1}}{\sum_{\ell \in I} \dfrac{\varphi(\beta_\ell^{j-1})}{\lambda_\ell}} & , \quad i \in I \\ \beta_i^{j-1} & , \quad i \notin I \end{cases} \qquad (53)$$

(53)

where $\Delta\beta_S^{j-1} = \beta_S^{j-1} - \beta_S^0$.

6. EXAMPLE

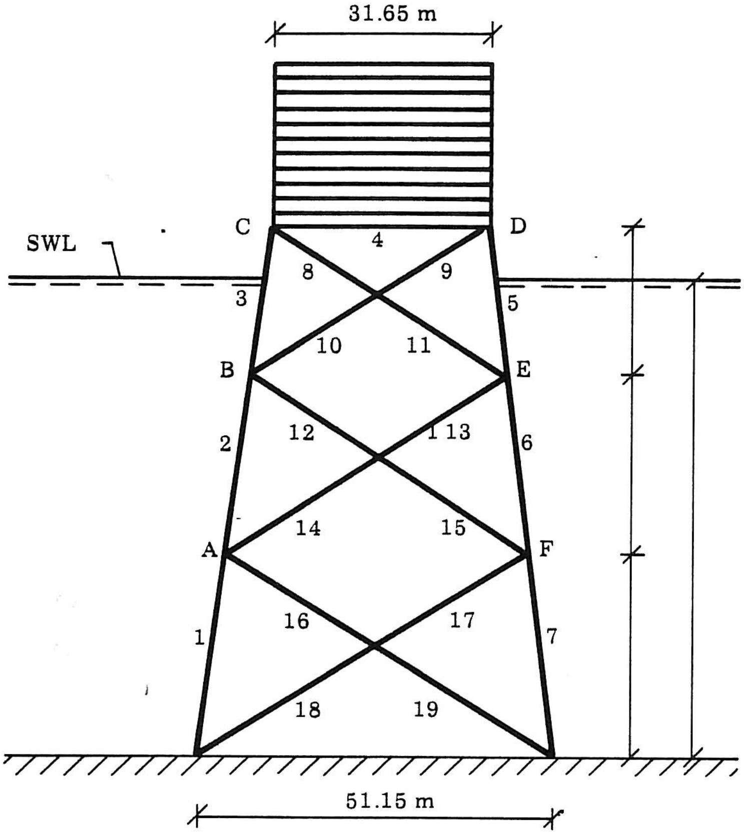
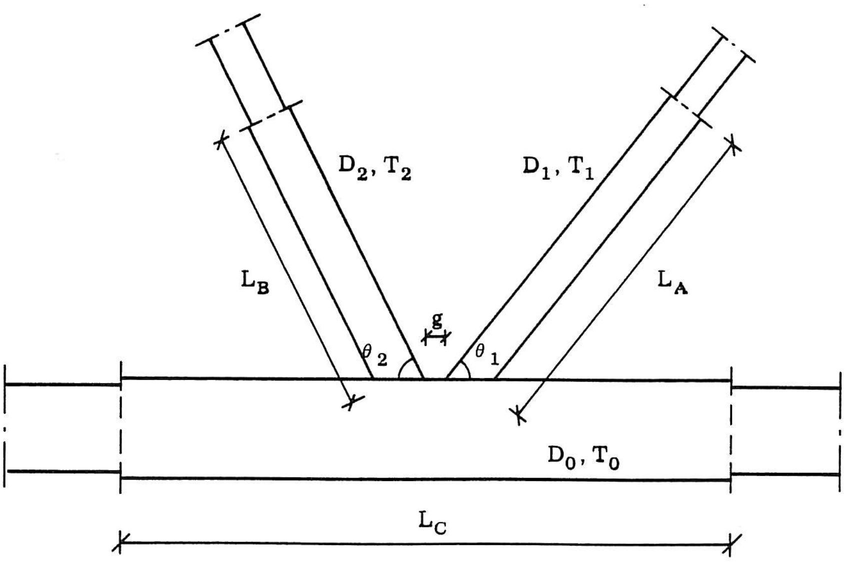


Figure 5. Offshore steel-jacket structure, see [27] for details.



$$\beta_i = \frac{D_i}{D_0}$$

$$\gamma = \frac{D_0}{2T_0}$$

$$\tau_i = \frac{T_i}{T_0}$$

$$\delta = \frac{g}{D_0}$$

Figure 6. K-joint E in jacket structure.

A K-joint in a plane model of a tubular steel-jacket offshore platform is considered, see figs. 5 and 6. The problem to find the optimal design of the K-joint under some reliability constraints is considered. The optimization variables, their limits and initial values are (dimension: [m]):

$t_1 = D_0$	$t_1^{\ell} = 1.80$	$t_1^{0} = 2.50$	$t_1^{u} = 3$
$t_2 = T_0$	$t_2^{\ell} = 0.03$	$t_2^{0} = 0.10$	$t_2^{u} = 0.2$
$t_3 = D_1$	$t_3^{\ell} = 0.80$	$t_3^{0} = 1.10$	$t_3^{u} = 1.5$
$t_4 = T_1$	$t_4^{\ell} = 0.02$	$t_4^{0} = 0.045$	$t_4^{u} = 0.2$
$t_5 = D_2$	$t_5^{\ell} = 0.80$	$t_5^{0} = 1.30$	$t_5^{u} = 1.5$
$t_6 = T_2$	$t_6^{\ell} = 0.02$	$t_6^{0} = 0.04$	$t_6^{u} = 0.2$

The limits are chosen so that connection with the adjacent tubular members is possible. Further, $L_A = L_B = 1.9$ m, $L_C = 5.4$ m, $\theta_1 = 50.3°$ and $\theta_2 = 63.9°$ are used.

Before stating the optimization problem the uncertain quantities are introduced. The load which consists of wave, wind and dead load is modelled by two independent random variables P_1 and P_2, see [30] for details. The reliability constraints are divided into four groups:

Ultimate yield capacity of beams (elasto-plastic material). The safety margins corresponding to the four cross-sections are written, see figure 7

$$M_i^Y = Z_1 - \left[\left| \frac{M_i}{M_{F_i}} \right| - \cos\left(\frac{\pi}{2} \frac{N_i}{N_{F_i}} \right) \right] \quad , \quad i = 1, 2, 3, 4 \tag{54}$$

where

$$N_{F_i} = \sigma_y \pi D_j T_j \quad , \quad i = 1, 2, 3, 4 \tag{55}$$

$$M_{F_i} = \sigma_y D_j^2 T_j \quad , \quad i = 1, 2, 3, 4 \tag{56}$$

$$N_i = a_{i1} P_1 + a_{i2} P_2 + a_{i3} \quad , \quad i = 1, 2, 3, 4 \tag{57}$$

$$M_i = b_{i1} P_1 + b_{i2} P_2 + b_{i3} \quad , \quad i = 1, 2, 3, 4 \tag{58}$$

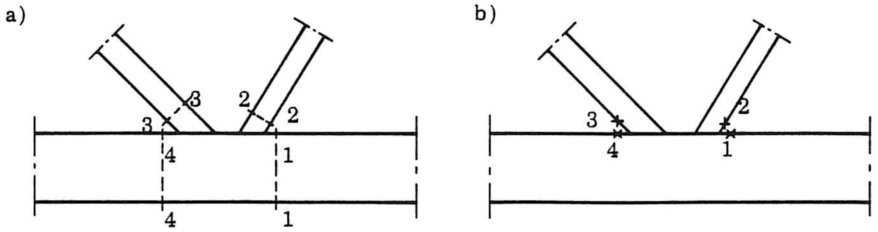


Figure 7. a) Cross-section numbering, b) numbering of hot spots.

σ_y and Z_1 are stochastic variables modelling the yield stress and the model uncertainty connected with the use of equation (54). $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are estimated by a linear elastic analysis of the whole structure:

$$\underset{\sim}{a} = \begin{bmatrix} -2299 & -31080 & -24.4 \text{ kN} \\ -835 & -29920 & -12.3 \text{ kN} \\ -1260 & -1212 & 119.6 \text{ kN} \\ 1329 & 939 & -28.8 \text{ kN} \end{bmatrix}$$

$$\underset{\sim}{b} = \begin{bmatrix} 96.8 \text{ m} & 271.4 \text{ m} & -136.4 \text{ kNm} \\ -276.8 \text{ m} & -648.3 \text{ m} & 227.3 \text{ kNm} \\ 215.8 \text{ m} & -323.3 \text{ m} & -55.0 \text{ kNm} \\ -35.8 \text{ m} & 700.1 \text{ m} & -35.8 \text{ kNm} \end{bmatrix}$$

$\underset{\sim}{a}$ and $\underset{\sim}{b}$ are assumed to be constant when optimizing the considered local joint.

Ultimate capacity of joint. The ultimate capacity of the joint is estimated using the empirical formula by Yura et al. [10, 30]. Each brace is considered separately. The safety margins corresponding to the two braces are written

$$M_1^u = Z_2 - \left| \frac{N_2}{Z_3 P_{u_1}} \right| - \left(\frac{|M_2|}{Z_4 M_{u_1}} \right)^{1.2} \tag{59}$$

$$M_2^u = Z_2 - \left| \frac{N_3}{Z_3 P_{u_2}} \right| - \left(\frac{|M_3|}{Z_4 M_{u_2}} \right)^{1.2} \tag{60}$$

where

$$P_{u_i} = \sigma_y T_0^2 (3.4 + 19\,\beta_i) Q_{g_i} / \sin\theta_i \quad , \quad i = 1, 2, \; 10 < \gamma < 48, \; 0.19 < \beta_i < 1 \tag{61}$$

$$M_{u_i} = 0.8\, D_i \sigma_y T_0^2 (3.4 + 19\,\beta_i) / \sin\theta_i \quad , \quad i = 1, 2, \; 9 < \gamma < 48, \; 0.19 < \beta < 1 \tag{62}$$

$$Q_{g_i} = \max\{1, \; 1.8 - \frac{0.8 \max\{0, g\}}{D_i}\} \tag{63}$$

N_2, N_3, M_2 and M_3 are given by (57) - (58). Z_2, Z_3 and Z_4 are model uncertainty variables.

Local stability of beams. Each of the four cross-sections in figure 7 is considered separately. The following safety margins can be stated using an API-based model, [18]:

$$M_i^S = Z_5 + \frac{\dfrac{N_i}{\pi D_j T_j} + \dfrac{4|M_i|}{\pi D_j^2 T_j}}{\sigma_y \left(1.64 - 0.274 \left(\dfrac{D_0}{2T_j}\right)^{0.25}\right)} \quad , \quad i = 1, 2, 3, 4 \tag{64}$$

where Z_5 is a model uncertainty variable. D_j and T_j are the diameter and thickness of the considered cross-section, respectively. N_i and M_i are given by (57) - (58).

Fatigue failure of joint. For simplicity the fatigue load is modelled as a stationary narrow-banded zero-mean Gaussian stochastic process with standard deviation σ_p and zero-upcrossing rate ν_p. The fatigue load is assumed to be independent of the permanent load P_2 and the extreme load P_1. To calculate the stress at the four critical points, see figures 5 and 7, we need the following stress concentration factors suggested empirically by Kuang, see [1]

$$SCF_i^a = 1.506\,\gamma^{0.666}\,\beta_i^{-0.59}\,\tau_i^{1.104}\,\delta^{0.067}\,\sin^{1.521}\theta_j \quad , \quad i = 1,4 \tag{65}$$

$$SCF_i^a = 0.920\,\gamma^{0.157}\,\beta_i^{-0.441}\,\tau_i^{0.560}\,\delta^{0.058}\,\exp(1.448\,\sin\theta_j) \quad , \quad i = 2,3 \tag{66}$$

$$SCF_i^m = 1.882\,\gamma^{0.38}\,\beta_i^{0.06}\,\tau_i^{0.94}\,\sin^{0.9}\theta_j \quad , \quad i = 1,4 \tag{67}$$

$$SCF_i^m = 2.827\,\beta_i^{-0.35}\,\tau_i^{0.35}\,\sin^{0.5}\theta_j \quad , \quad i = 2,3 \tag{68}$$

where $\theta_j = \theta_1$ if i = 1, 2 and $\theta_j = \theta_2$ if i = 3, 4. For the parameter δ the following rule is used, [1]

$$\delta = \max\{\delta, \ 0.01\} \tag{69}$$

(65) - (68) are valid within certain limits of β, γ, and τ, see [1].

The stresses at the four points can now be estimated

$$\sigma_i = SCF_i^a\,\sigma_{N_i} + SCF_i^m\,\sigma_{M_i} \quad , \quad i = 1,3 \tag{70}$$

$$\sigma_i = SCF_i^a\,\sigma_{N_i} - SCF_i^m\,\sigma_{M_i} \quad , \quad i = 2,4 \tag{71}$$

where σ_{N_i} and σ_{M_i} are the elastic nominal stresses from the axial load and the moment load if $P_1 = 1$ and $P_2 = 0$.

The fatigue safety margins at the four points can be written as follows, if the linear damage accumulation model (Miner's rule) is used, [29]

$$M_i^F = \Delta - \frac{T_L \nu_p B^m}{K}\left(\frac{t_j}{32}\right)^{m_1}(2\sqrt{2})^m(\sigma_i\sigma_p)^m\,\Gamma\left(1+\frac{m}{2}\right) \quad , \quad i = 1,2,3,4 \tag{72}$$

where

T_L: expected lifetime. Here we use T_L = 25 years

ν_p: zero-upcrossing rate. Here we use $\nu_p = 0.2\ s^{-1}$

σ_p: standard deviation of fatigue load. Here we use σ_p = 1 N.

Γ: Gamma function

t_j: $= T_0$ if i = 1, 4, $= T_1$ if i = 2, and $= T_2$ if i = 3 (T_i in [mm]). If $T_i < 32$ mm $t_j = 32$

Δ: model uncertainty variable which model the uncertainty connected by using Miner's rule as a measure of damage accumulation

m: constant in S-N relation used in Miner's rule. Here m = 3

K: constant in S-N relation used in Miner's rule. K is modelled as a random variable

m_1: random variable

B: model uncertainty variable which models the uncertainty connected with estimating the fatigue load, e.g. a) estimation of hot spot stress concentration factors (SCF), b) description of sea states, c) estimation of wave load (Morison's equation), d) determination of local forces in the structural system (mechanical model and analysis) and e) fabrication and assembling of the structure.

In table 1 the statistical characteristics of the 12 basic variables in the 14 safety margins (54), (59) - (60), (64) and (72) are shown. The basic variables are assumed to be independent.

basic variable	variable	distribution	expected value	standard deviation
X_1	P_1	EX1	1.711 kN	0.631 kN
X_2	P_2	N	1 kN	0.05 kN
X_3	σ_y	LN	$340\cdot10^3$ kNm^{-2}	$34\cdot10^3$ kNm^{-2}
X_4	Z_1	N	0.	0.05
X_5	Z_2	N	1.	0.05
X_6	Z_3	LN	1.161	0.2113
X_7	Z_4	LN	1.227	0.1706
X_8	Z_5	N	1.	0.05
X_9	Δ	N	1.	0.2
X_{10}	K	LN	$5.39\cdot10^{15}$ N^3 m^{-6}	$3.35\cdot10^{15}$ N^3 m^{-6}
X_{11}	B	LN	0.7	0.35
X_{12}	m_1	Ln	0.75	0.0375

Table 1. Statistical characteristics (EX1: extreme type 1, N: normal, LN: log-normal).

Two reliability-based optimization problems corresponding to (18) and (29) are considered. The first problem is based on formulation F4:

$$\text{minimize } F(\underset{\sim}{t}) = \rho\pi(L_C t_1 t_2 + L_A t_3 t_4 + L_B t_5 t_6) \tag{73}$$

$$\text{s.t.} \quad \beta_i(\underset{\sim}{t}) = \beta_i^Y(\underset{\sim}{t}) \geq \beta_i^0 \quad , \quad i = 1, 2, 3, 4 \tag{74}$$

$$\beta_{4+i}(\underset{\sim}{t}) = \beta_i^U(\underset{\sim}{t}) \geq \beta_{4+i}^0 \quad , \quad i = 1, 2 \tag{75}$$

$$\beta_{6+i}(\underset{\sim}{t}) = \beta_i^S(\underset{\sim}{t}) \geq \beta_{6+i}^0 \quad , \quad i = 1, 2, 3, 4 \tag{76}$$

$$\beta_{10+i}(\underset{\sim}{t}) = \beta_i^F(\underset{\sim}{t}) \geq \beta_{10+i}^0 \quad , \quad i = 1, 2, 3, 4 \tag{77}$$

$$t_1 - t_3 \geq 0 \tag{78}$$

$$t_1 - t_5 \geq 0 \tag{79}$$

$$\underset{\sim}{t}^{\ell} \leq \underset{\sim}{t} \leq \underset{\sim}{t}^u \tag{80}$$

In (73) ρ = 7.85 tm^{-3} and L_A, L_B, L_C are defined in figure 6. The reliability indices in (74) - (77) correspond to the above 14 safety margins. (78) and (79) ensure that the diameters of the braces are less than the diameter of the chord.

In figure 8 the iteration history obtained using procedure PE1 and β_i^0 = 3.0 is shown.

Next consider the systems reliability index based optimization problem (18). β_S is estimated using the above 14 safety margins in (7) and (9) (h = 14).

$$\text{minimize } F(\underset{\sim}{t}) = \rho\pi(L_C t_1 t_2 + L_A t_3 t_4 + L_B t_5 t_6) \tag{81}$$

$$\text{s.t.} \quad \beta_S(\underset{\sim}{t}) \geqslant \beta_S^0 \tag{82}$$

$$t_1 - t_3 \geqslant 0 \tag{83}$$

$$t_1 - t_5 \geqslant 0 \tag{84}$$

$$\underset{\sim}{t}^{\ell} \leqslant \underset{\sim}{t} \leqslant \underset{\sim}{t}^{u} \tag{85}$$

This problem is solved using the four procedures described above. The results are shown in figures 9 - 11 for β_S^0 = 3. Procedure PS1 where the gradients are estimated by finite differences converges to F = 10.00. Procedures PS2 and PS3 both converge to F = 10.01 with almost identical iteration histories. In procedure PS4 a sequence of problems of the form (68) - (80) is solved. PS4 converges to F = 10.11. In table 2 the results of PE1, PS1, PS2, PS3, and PS4 are shown.

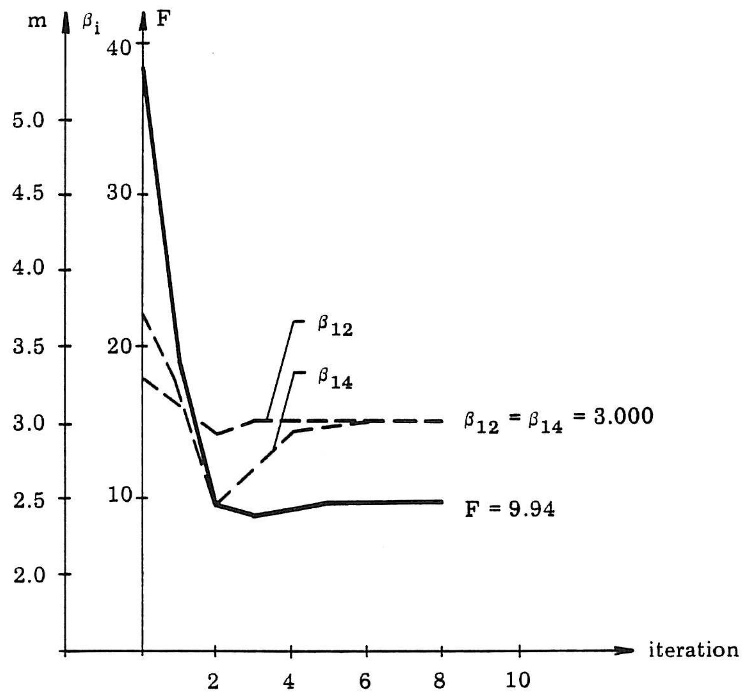


Figure 8. Procedure PE1.

procedure	t_1	t_2	t_3	t_4	t_5	t_6	F	sec*	iterations
PE1	1.80	0.0305	1.01	0.0200	1.80	0.0200	9.94	17	8
PS1	1.80	0.0311	1.01	0.0200	1.71	0.0200	10.00	152	10
PS2	1.80	0.0313	1.01	0.0200	1.66	0.0200	10.01	23	10
PS3	1.80	0.0311	1.01	0.0200	1.71	0.0200	10.01	37	10
PS4	1.80	0.0309	1.09	0.0200	1.80	0.0200	10.11	151	7

Table 2. Results of optimizations. t_i in [m] and F in [tons]. *) CDC Cyber 170-730.

From table 2 we see that the optimal values of t_1, t_4, and t_6 are at their lower bounds. As expected the conservative procedure PS4 gives a higher value of the objective function than PS1 (1% greater). The runs with PS2 and PS3 show that in this example the approximations (43) and (44) give sufficiently accurate estimates of the gradients of the systems reliability index β_S. If the results of PS2 and PS3 are compared with those of PS1 it is seen that the results of PS3 are closest to the »exact» results of PS1, but the amount of calculations in PS3 is much higher than that in PS2 (computer time 60% greater) due to the extra evaluations of $\Phi_{h_S}(\underset{\sim}{\beta}'_S ; \underset{\sim}{\bar{\rho}})$. From the table it is also seen that PS4 in this example requires much more computer time than PS2 and PS3.

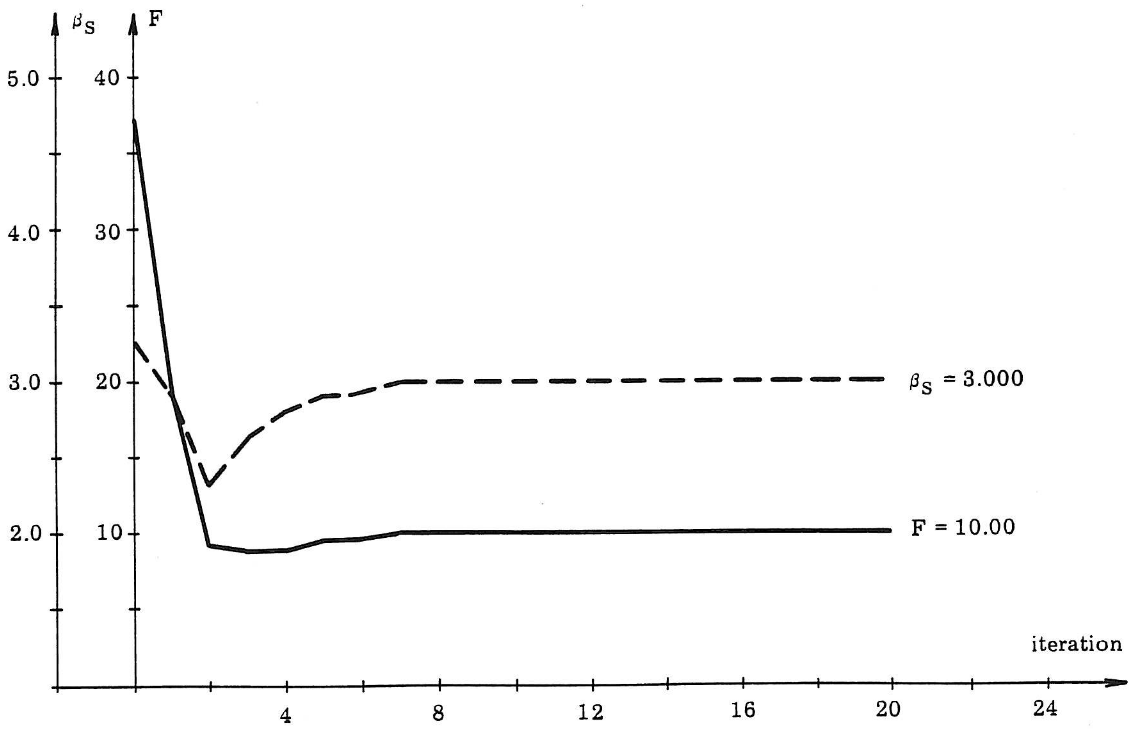


Figure 9. Procedure PS1.

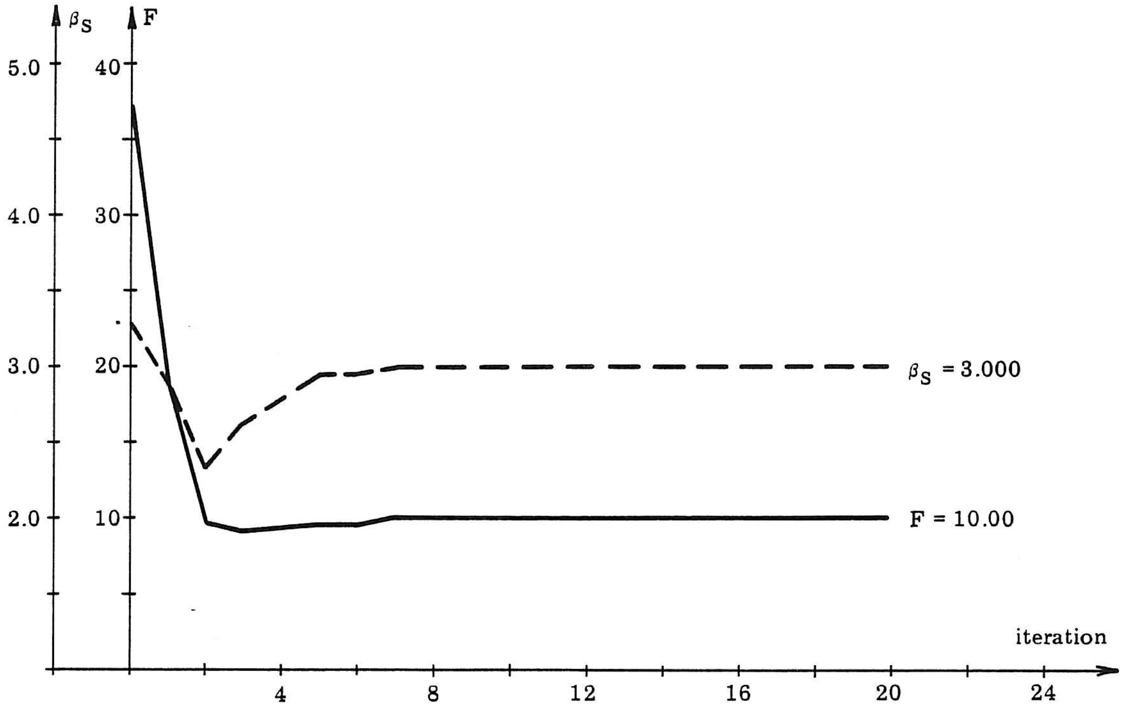


Figure 10. Procedure PS2 and PS3 (no visible difference).

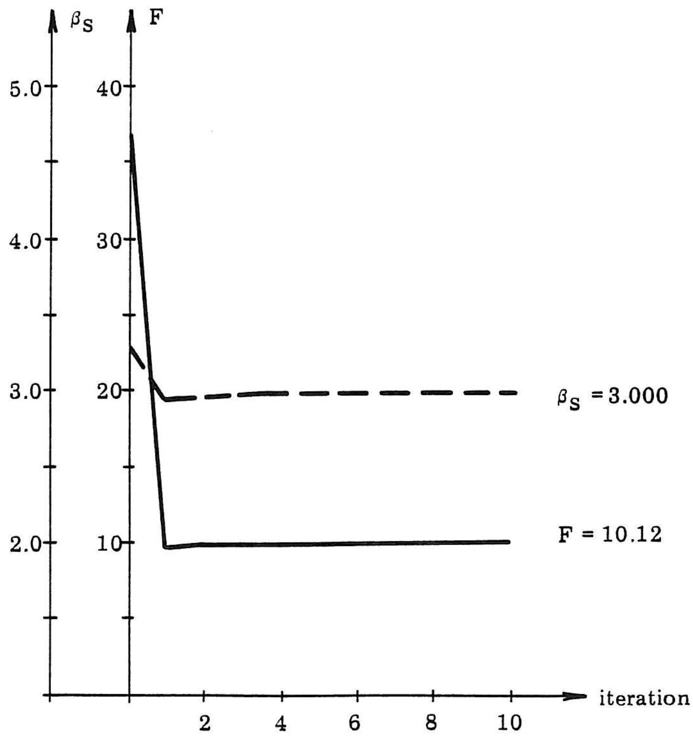


Figure 11. Procedure PS4.

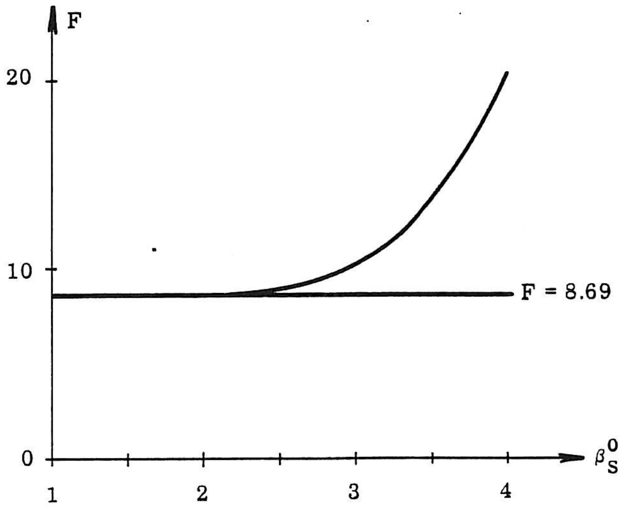


Figure 12.

In figure 12 the optimal weight is shown as a function of β_S^0. Procedure PS4 is used to solve the optimization problems. The lower bound F = 8.69 tons corresponds to the lower bound of the optimization variables.

The sensitivity of the reliability indices β_i, i = 1, 2, . . . , 14 and β_S with respect to variations in the statistical parameters can be estimated using (47) and (48). For β_S^0 = 3 the result of such an analysis at the optimal point $\underset{\sim}{t}^*$ is shown in table 3. From the table it appears that the standard deviation of the model uncertainty variable B has a very high sensitivity coefficient with respect to β_S, e.g. a reduction of $\sigma[B]$ from 0.35 to 0.30 would increase the systems reliability index β_S to approximately $\beta_S \sim 3.25$.

variable	parameter	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_S
P_1	$E[X_1]$	−0.43	−0.27	−0.32	0	−0.48	−0.10	−0.39	−0.07	−0.35	0	0	0	0	0	−0.04
	$\sigma[X_1]$	−1.35	−0.35	−4.58	0	−2.63	−0.65	−0.74	−0.10	−4.25	−1.21	0	0	0	0	−0.16
P_2	$E[X_2]$	−5.52	−7.23	−0.11	−0.13	−0.27	−0.09	−6.12	−1.55	0.20	−0.34	0	0	0	0	−0.25
	$\sigma[X_2]$	−6.30	−13.5	0	−0.02	−0.01	−0.01	−7.49	−2.37	−0.01	−0.11	0	0	0	0	−0.29
σ_y	$E[X_3]$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\sigma[X_3]$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Z_1	$E[X_4]$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\sigma[X_4]$	−4.92	−8.53	−1.79	−1.64	0	0	0	0	0	0	0	0	0	0	−0.08
Z_2	$E[X_5]$	0	0	0	0	2.73	0.69	0	0	0	0	0	0	0	0	0.12
	$\sigma[X_5]$	0	0	0	0	−1.44	−0.47	0	0	0	0	0	0	0	0	−0.07
Z_3	$E[X_6]$	0	0	0	0	2.82	0.82	0	0	0	0	0	0	0	0	0.13
	$\sigma[X_6]$	0	0	0	0	−3.84	−1.38	0	0	0	0	0	0	0	0	−0.17
Z_4	$E[X_7]$	0	0	0	0	0.23	0.01	0	0	0	0	0	0	0	0	0.01
	$\sigma[X_7]$	0	0	0	0	−0.06	0	0	0	0	0	0	0	0	0	0
Z_5	$E[X_8]$	0	0	0	0	0	0	8.23	2.03	3.38	19.8	0	0	0	0	0.21
	$\sigma[X_8]$	0	0	0	0	0	0	−13.6	−4.09	−3.17	−3.92	0	0	0	0	−0.35
Δ	$E[X_9]$	0	0	0	0	0	0	0	0	0	0	0.12	0.11	0.13	0.70	0.67
	$\sigma[X_9]$	0	0	0	0	0	0	0	0	0	0	−0.06	−0.05	−0.07	−0.30	−0.28
K	$E[X_{10}]$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\sigma[X_{10}]$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B	$E[X_{11}]$	0	0	0	0	0	0	0	0	0	0	0.07	0	0.12	−0.08	−0.08
	$\sigma[X_{11}]$	0	0	0	0	0	0	0	0	0	0	−1.08	−0.86	−1.24	−5.36	−5.09
m_1	$E[X_{12}]$	0	0	0	0	0	0	0	0	0	0	0	0.05	0.06	0.02	0.02
	$\sigma[X_{12}]$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3. Sensitivity coefficients. E[•] and σ[•] signify expected value and standard deviation.

7. CONCLUSIONS

The first order reliability method to estimate the reliability of structural elements is briefly described. Next, reliability-based structural optimization problems are formulated and discussed. Especially formulations based on element reliability indices and systems reliability indices are considered. From an optimization point of view optimization of structural elements is considered.

Two methods are proposed for efficient estimation of the gradient of the systems reliability index with respect to the optimization variables. In an example both methods are shown to be very effective when compared with estimation of gradients by finite differences.

Five optimization procedures to solve the reliability-based structural optimization problems are proposed. Especially to solve the systems reliability index based optimization problem a method is suggested which solves a sequence of easily solved element reliability index based optimization problems. The method uses Lagrange multiplier estimates to adjust the element reliability index constraints.

In an example a tubular K-joint in an offshore platform is optimized with reliability constraints based on yielding, stability, punching and fatigue failure modes. All the proposed procedures converge. The most effective procedure for systems reliability index based problems is in this example a procedure where the system gradients are estimated by a simple approximation.

In the example the NLPQL algorithm (a sequential quadratic optimization algorithm with augmented Lagrangian type line search) is used to solve the mathematical optimization problems. Use of this algorithm in the proposed optimization procedures has in this example shown to give good results.

8. REFERENCES

[1] Almar-Næs, A. (ed.): *Fatigue Handbook.* Tapir, Trondheim, 1985

[2] Bennett, R. M.: *Reliability Analysis of Frame Structures with Brittle Components.* Structural Safety, Vol. 2, 1985, pp. 281-290.

[3] Bjerager, P.: *Reliability Analysis of Structural Systems.* Ph.D. Thesis, Department of Structural Engineering, Technical University of Denmark, Series R, No. 183, 1984.

[4] Burns, S. A. & S. Ramamurthy: *Mathematical Programming in Structural Design.* ASCE, Journal of Structural Engineering, Vol. 109, No. 7, 1983, pp. 1669-1679.

[5] Casciati, F. & L. Faravelli: *Structural Reliability and Structural Design Optimization.* ICOSSAR'85, Kobe, Japan, Vol. III, 1985, pp. 61-70.

[6] Cornell, C. A.: *Probability-Based Structural Code.* American Concrete Institute, ACI Journal, Vol. 100, 1969, pp. 974-985.

[7] Ditlevsen, O.: *Basic Reliability Concepts.* Nato Advanced Study Institute (ed. P. Thoft-Christensen), Martinus Nijhoff Publishers, The Netherlands, 1983, pp. 1 - 56.

[8] Frangopol, D. M.: *Sensitivity of Reliability-Based Optimum Design.* ASCE, Journal of Structural Engineering, Vol. 111, No. 8, 1985.

[9] Hasofer, A. M. & N. C. Lind: *An Exact and Invariant First-Order Reliability Format.* ASCE, Journal of the Engineering Mechanics Division, Vol. 100, No. EM1, 1974, pp. 111-121.

[10] Hoadley, P. E. & J. A. Yura: *Ultimate Strength of Tubular Joints Subjected to Combined Loads.* University of Texas, Austin, 1983.

[11] Hohenbichler, M.: *An Approximation to the Multivariate Normal Distribution.* Proc. EUROMECH 155, Reliability Theory of Structural Engineering Systems, DIALOG 6-82, Danish Engineering Academy, Lyngby, 1982, pp. 79-100.

[12] Hohenbichler, M.: *Mathematische Grundlagen der Zuverlässigkeitsmethode erster Ordnung und einige Erweiterungen.* Thesis, Technical University of Munich, W. Germany, 1984.

[13] Johnson, N. L. & S. Kotz: *Distributions in Statistics. Continuous Multivariate Distributions.* John Wiley & Sons, 1972.

[14] Kirsch, U.: *An Improved Multilevel Structural Synthesis Method.* Journal of Structural Mechanics, Vol. 13, No. 2, 1985, pp. 123-144.

[15] Koski, J. & R. Silvennoinen: *Pareto Optima of Isostatic Trusses.* Computer Methods in Applied Mechanics and Engineering, Vol. 31, 1982, pp. 265-279.

[16] Madsen, H. O., S. Krenk & N. C. Lind: *Safety of Structures.* Printice-Hall, 1986.

[17] Melchers, R. E. & L. K. Tang: *Dominant Failure Modes in Stochastic Structural Systems.* Structural Safety, Vol. 2, 1984, pp. 127-143.

[18] Miller, C. D.: *Buckling Design Methods for Steel Structures - A State-of-the-Art.* In Faulkner, D. et al.: Integrity of Offshore Structures, Applied Science Publishers, London, 1981, pp. 397-418.

[19] Murotsu, Y., H. Okada, M. Yonezawa & M. Kishi: *Identification of Stochastically Dominant Failure Modes in Frame Structure.* 4th Int. Conf. on Appl. Stat. and Prob. in Soil and Struct. Eng., Firenze, 1983, pp. 1325-1338.

[20] Murotsu, Y., M. Kishi, H. Okada, M. Yonezawa & K. Taguchi: *Probabilistically Optimum Design of Frame Structure.* In P. Thoft-Christensen (ed).: System Modelling and Optimization, Springer-Verlag, Berlin, 1984, pp. 545-554.

[21] Schittkowski, K.: *Theory, Implementation, and Tests of a Non-Linear Programming Algorithm.* Proc. EUROMECH 164 on »Optimization Methods in Structural Design«, Bibl. Institut, Mannheim, 1983, pp. 122-132.

[22] Stahl, B. & J. F. Geyer: *Fatigue Reliability of Parallel Member Systems.* ASCE, Journal of Structural Engineering, Vol. 110, No. 10, 1984, pp. 2307-2323.

[23] Sørensen, J. D. & P. Thoft-Christensen: *Recent Advances in Optimal Design of Structures from a Reliability Point of View.* 9th ARTS, Bradford, 1986.

[24] Sørensen, J. D. & P. Thoft-Christensen: *Structural Optimization with Reliability Constraints.* Proc. 12th IFIP Conf. on »System Modelling and Optimization», Springer-Verlag, 1986.

[25] Thoft-Christensen, P. & M. J. Baker: *Structural Reliability Theory and Its Applications.* Springer-Verlag, Berlin, 1982.

[26] Thoft-Christensen, P. & J. D. Sørensen: *Calculation of Failure Probabilities of Ductile Structures by the β-Unzipping Method.* Institute of Building Technology and Structural Engineering, The University of Aalborg, 1982.

[27] Thoft-Christensen, P.: *Structural Reliability Theory.* Proc. ESRA Pre-Launching Meeting, Ispra, Italy, 1985, pp. 82-99.

[28] Vanderplaats, G. N.: *Numerical Optimization Techniques for Engineering Design.* McGraw-Hill, N.Y., 1984.

[29] Wirshing, P. H.: *Fatigue Reliability for Offshore Structures.* ASCE, Journal of Structural Engineering, Vol. 110, No. 10, 1984, pp. 2340-2356.

[30] Yura, J. A., N. Zettlemayer & I. F. Edwards: *Ultimate Capacity Equations for Tubular Joints.* Offshore Technology Conference, Houston, Texas, OTC 3690, 1980, pp. 113-125.

APPENDIX

Determination of $\partial\Phi_n/\partial\beta_i$ and $\partial\Phi_n/\partial\rho_{ij}$

The normal distribution function Φ_n is defined as

$$\Phi_n(\underset{\sim}{\beta};\underset{\sim}{\rho}) = \int_{-\infty}^{\beta_1}\int_{-\infty}^{\beta_2}\cdots\int_{-\infty}^{\beta_n}\varphi_n(\underset{\sim}{x};\underset{\sim}{\rho})dx_1dx_2\ldots dx_n \tag{A1}$$

where φ_n is the normal density function

$$\varphi_n(\underset{\sim}{x};\underset{\sim}{\rho}) = \frac{1}{(2\pi)^{n/2}|\underset{\sim}{\rho}|^{1/2}}\exp(-\frac{1}{2}\underset{\sim}{x}^T\underset{\sim}{\rho}^{-1}\underset{\sim}{x}) \tag{A2}$$

In order to determine $\partial\Phi_n/\partial\beta_i$ we exchange index i and n so that $\underset{\sim}{\rho}$, $\underset{\sim}{x}$, and $\underset{\sim}{\beta}$ can be partitioned as, [13]

$$\left[\begin{array}{c|c}\underset{\sim}{\rho}_{11} & \underset{\sim}{\rho}_{12}\\ \hline \underset{\sim}{\rho}_{12}^T & 1\end{array}\right], \quad \left[\begin{array}{c}\underset{\sim}{x}_1\\ \hline x_i\end{array}\right] \quad \text{and} \quad \left[\begin{array}{c}\underset{\sim}{\beta}_1\\ \hline \beta_i\end{array}\right]$$

$\underset{\sim}{x}^T\underset{\sim}{\rho}^{-1}\underset{\sim}{x}$ and $|\underset{\sim}{\rho}|$ can now be written

$$\underset{\sim}{x}^T\underset{\sim}{\rho}^{-1}\underset{\sim}{x} = (\underset{\sim}{x}_1-\underset{\sim}{\mu}_1)^T\underset{\sim}{r}_{11}^{-1}(\underset{\sim}{x}_1-\underset{\sim}{\mu}_1)+x_i^2 \tag{A3}$$

$$|\underset{\sim}{\rho}| = |\underset{\sim}{r}_{11}| \tag{A4}$$

where

$$\underset{\sim}{r}_{11} = \underset{\sim}{\rho}_{11}-\underset{\sim}{\rho}_{12}\underset{\sim}{\rho}_{12}^T \tag{A5}$$

$$\underset{\sim}{\mu}_1 = \underset{\sim}{\rho}_{12}x_i \tag{A6}$$

Therefore, $\varphi_n(\underset{\sim}{x};\underset{\sim}{\rho})$ can be written

$$\varphi_n(\underset{\sim}{x};\underset{\sim}{\rho}) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}x_i^2)\frac{1}{(2\pi)^{\frac{n-1}{2}}|\underset{\sim}{r}_{11}|}\exp(-\frac{1}{2}(\underset{\sim}{x}_1-\underset{\sim}{\mu}_1)^T\underset{\sim}{r}_{11}^{-1}(\underset{\sim}{x}_1-\underset{\sim}{\mu}_1)) \tag{A7}$$

$\frac{\partial\Phi_n}{\partial\beta_i}$ then becomes

$$\frac{\partial\Phi_n}{\partial\beta_i} = \varphi(\beta_i)\Phi_{n-1}(\underset{\sim}{\beta}';\underset{\sim}{\rho}') \tag{A8}$$

where

$$\beta_j' = \frac{\{\underset{\sim}{\beta}_1\}_j - \{\underset{\sim}{\rho}_{12}\}_j \beta_i}{\sqrt{\{\underset{\sim}{r}_{11}\}_{jj}}} \tag{A9}$$

$$\rho_{jk}' = \frac{\{\underset{\sim}{r}_{11}\}_{jk}}{\sqrt{\{\underset{\sim}{r}_{11}\}\{\underset{\sim}{r}_{11}\}_{kk}}} \tag{A10}$$

$\{\underset{\sim}{a}\}_j$ symbolizes element j in the matrix $\underset{\sim}{a}$.

To determine $\partial\Phi_n/\partial\rho_{ij}$ we exchange the indices (i, j) with (n, n − 1) so that $\underset{\sim}{\rho}$, $\underset{\sim}{x}$ and $\underset{\sim}{\beta}$ can be partitioned as

$$\left[\begin{array}{c|cc} \underset{\sim}{\rho}_{11} & \multicolumn{2}{c}{\underset{\sim}{\rho}_{12}} \\ \hline \underset{\sim}{\rho}_{12}^T & 1 & \rho_{ij} \\ & \rho_{ij} & 1 \end{array}\right] = \left[\begin{array}{c|c} \underset{\sim}{\rho}_{11} & \underset{\sim}{\rho}_{12} \\ \hline \underset{\sim}{\rho}_{12}^T & \underset{\sim}{\rho}_{22} \end{array}\right]$$

$$\left[\begin{array}{c} \underset{\sim}{x}_1 \\ \hline x_i \\ x_j \end{array}\right] = \left[\begin{array}{c} \underset{\sim}{x}_1 \\ \hline \underset{\sim}{x}_2 \end{array}\right]$$

$$\left[\begin{array}{c} \underset{\sim}{\beta}_1 \\ \hline \beta_i \\ \beta_j \end{array}\right] = \left[\begin{array}{c} \underset{\sim}{\beta}_1 \\ \hline \underset{\sim}{\beta}_2 \end{array}\right]$$

$\underset{\sim}{x}^T \underset{\sim}{\rho}^{-1} \underset{\sim}{x}$ and $\underset{\sim}{\rho}$ can now be written

$$\underset{\sim}{x}^T \underset{\sim}{\rho}^{-1} \underset{\sim}{x} = (\underset{\sim}{x}_1 - \underset{\sim}{\mu}_1)^T \underset{\sim}{r}_{11}^{-1} (\underset{\sim}{x}_1 - \underset{\sim}{\mu}_1) + \underset{\sim}{x}_2^T \underset{\sim}{\rho}_{22}^{-1} \underset{\sim}{x}_2 \tag{A11}$$

$$|\underset{\sim}{\rho}| = |\underset{\sim}{r}_{11}|\,|\underset{\sim}{\rho}_{22}| \tag{A12}$$

where

$$\underset{\sim}{r}_{11} = \underset{\sim}{\rho}_{11} - \underset{\sim}{\rho}_{12} \underset{\sim}{\rho}_{22}^{-1} \rho_{12}^T \tag{A13}$$

$$\underset{\sim}{\mu}_1 = \underset{\sim}{\rho}_{12} \underset{\sim}{\rho}_{22}^{-1} \underset{\sim}{x}_2 \tag{A14}$$

Therefore, $\varphi_n(\underset{\sim}{x}\,;\underset{\sim}{\rho})$ can be written

$$\varphi_n(\underset{\sim}{x}\,;\underset{\sim}{\rho}) = \frac{1}{2\pi|\underset{\sim}{\rho}_{22}|} \exp(-\frac{1}{2}\underset{\sim}{x}_2^T \underset{\sim}{\rho}_{22}^{-1} \underset{\sim}{x}_2) \frac{1}{(2\pi)^{\frac{n-2}{2}} |\underset{\sim}{r}_{11}|} \exp(-\frac{1}{2}(\underset{\sim}{x}_1 - \underset{\sim}{\mu}_1)^T \underset{\sim}{r}_{11}^{-1} (\underset{\sim}{x}_1 - \underset{\sim}{\mu}_1)) \tag{A15}$$

$\dfrac{\partial \Phi_n}{\partial \rho_{ij}}$ then becomes

$$\frac{\partial \Phi_n}{\partial \rho_{ij}} = \int_{-\infty}^{\beta_1} \cdots \int_{-\infty}^{\beta_i} \cdots \int_{-\infty}^{\beta_j} \cdots \int_{-\infty}^{\beta_n} \frac{\partial \varphi_n(\underset{\sim}{x}\,;\underset{\sim}{\rho})}{\partial \rho_{ij}}\, dx_1 \ldots dx_i \ldots dx_j \ldots dx_n$$

$$= \int_{-\infty}^{\beta_1} \cdots \int_{-\infty}^{\beta_i} \cdots \int_{-\infty}^{\beta_j} \cdots \int_{-\infty}^{\beta_n} \frac{\partial^2 \varphi_n(\underset{\sim}{x}\,;\underset{\sim}{\rho})}{\partial x_i \partial x_j}\, dx_1 \ldots dx_i \ldots dx_j \ldots dx_n$$

$$= \int_{-\infty}^{\beta_1} \cdots \int_{-\infty}^{\beta_n} \varphi_n(x_1, \ldots, \beta_i, \ldots, \beta_j, \ldots, \beta_n\,;\underset{\sim}{\rho})\, dx_1 \ldots dx_n$$

$$= \varphi_2(\beta_i, \beta_j; \rho_{ij})\, \Phi_{n-2}(\underset{\sim}{\beta}''\,;\underset{\sim}{\rho}'') \tag{A16}$$

where

$$\beta_k'' = \frac{\{\underset{\sim}{\beta}_1\}_k - \{\underset{\sim}{\rho}_{12}\, \underset{\sim}{\rho}_{22}^{-1}\, \underset{\sim}{\beta}_2\}_k}{\sqrt{\{\underset{\sim}{r}_{11}\}_{kk}}} \tag{A17}$$

$$\rho_{k\ell}'' = \frac{\{\underset{\sim}{r}_{11}\}_{k\ell}}{\sqrt{\{\underset{\sim}{r}_{11}\}_{kk}\{\underset{\sim}{r}_{11}\}_{\ell\ell}}} \tag{A18}$$

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